

Equipping Teachers for Inclusive Science Technology Engineering and Mathematics Methodologies for Sustainable Development

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Abstract

Persistent gender inequalities abound in the teaching and learning of Science, Technology, Engineering and Mathematics (STEM), particularly, but not only in African countries. While these inequalities stem from broader cultural and social factors, and are not predominantly the result of teaching in schools, teachers can play a strong role in supporting all learners, particularly girls. To perform well in STEM, girls need to develop a passion for STEM subjects and to choose to study STEM and develop STEM careers. It is argued that developing inclusive teaching methodologies is necessary but not sufficient in mathematics classrooms. To develop classrooms where all learners feel supported to succeed in mathematics, STEM requires shifts in teachers' beliefs, practices and dispositions about who can and cannot learn mathematics. This paper outlines some of the persisting gender disparities in mathematics and STEM, it discusses challenges in achievement and affect in mathematics and STEM, and develops the argument that teachers need to strongly believe and communicate that all learners can do mathematics, to high levels. This argument is developed through reflecting on the researcher's two recent research projects: the first is on helping teachers to work more productively with learner errors in mathematics and, the second, on learners' mathematical identities.

Word Count: 203

Keywords: *Inclusive STEM, Methodologies, Sustainable Development, Inequalities.*

Gender Challenges in Mathematics Education

With some slight variations, gender parity in achievement in mathematics has been reached in richer countries, as measured by the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) studies. This is the case even at the top end of the achievement scale. However, recently there has been some slippage of girls in Australia suggesting that equality has to be continually monitored and achieved. In Africa, boys still achieve somewhat higher than girls, based on the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) and the Analysis Programme of the CONFEMEN Education Systems PASEC studies with some notable exceptions. For example, in SACMEQ girls have attained equality with boys in Lesotho, Namibia and Botswana but are slightly better in Mauritius, Seychelles and South Africa. It is important to be careful when interpreting these results because sometimes “equal” means “equally low”. For example, in South Africa, the mean score for mathematics for girls in SACMEQ III was 510 (for boys it was 480), while in Kenya, the girls' mean score was 545 (for boys, 570). So, girls in South Africa are not performing as well as girls in Kenya, even though they are more “equal”.

The results for attitudes and affect towards mathematics are somewhat different, showing that even when girls' achievement is as good as boys', their attitudes are less positive. Boys show higher self-confidence and positive attitudes while girls show more mathematics anxiety. A recent paper outlined a “gender equality paradox”, showing that even when girls achieved equally with boys in science and mathematics in wealthier countries and were more capable of college level study—

they were less likely to choose to study STEM subjects in further studies. This was because they achieved relatively better in reading than in science and mathematics (based on PISA results), and were more likely to choose careers based on their academic strengths. The study shows that “paradoxically, the sex differences in the magnitude of relative academic strengths and the pursuit of STEM degrees rose with increase in national gender equality” Stoet and Geary, 2018 (p. 581).

A number of explanations for these differences have been given in the above studies. For SACMEQ, the role of female teachers and principals is important because they can act as role models. However, Dickerson et. al. (2015) shows that the number of female teachers does not have a significant effect on achievement outcomes in African contexts. They show that broader cultural factors, in particular, the empowerment of women, explain gender disparities, rather than any immediate school effects (p. 15-16). These broader cultural factors include the status of women in society, violence against women and girls and poverty. Issues such as socio-economic status and race are also important. For example, in South Africa, it is mainly poor black students who achieve low results in STEM. Stoet and Geary suggest that the “gender equality paradox” may be the result of a wider range of opportunities available to women in more ‘equal’ countries whereas STEM careers are more likely to support equality for women in less equal societies. Given these arguments, it is clear that, while female and male teachers can become better at developing inclusive methodologies for boys and girls, real change will only come when the broader society and culture takes the empowerment of women seriously. As teachers and teacher-educators, we can and must contribute to this broader empowerment.

Moreover, as suggested above, the challenges go beyond gender-equality, because in many countries, both girls and boys perform poorly. For instance, in South Africa, the achievements on the Annual National Assessments decrease substantially as learners progress through the system, with the average result in 2014 going from 68% in grade 1 to 56% in grade 3, to 43% in grade 6 and 11% in grade 9. Learners' difficulties in these tests include algebraic and spatial relationships, problem-solving, and logical reasoning, i.e. they do not develop the higher-order mathematical skills required for success in STEM. Reasons given for learners' poor performance include: teachers' weak subject knowledge (a favourite reason currently in South Africa) and learners' difficulties in the language of instruction, which is English (This is not the main language of most learners). The broader societal reasons such as violence, poverty and inequality in schools and in society are not being discussed frequently enough, especially by those of us who work in the areas of curriculum and pedagogy; but they are crucial. Also important are issues such as increasing assessment in schools, which produces increased mathematics anxiety, teachers' and parents' beliefs about who can or cannot do mathematics. Teacher demoralization also plays out strongly in many schools in South Africa.

Towards Research-based Solutions

Researchers such as Jo Boaler (2016) argue that all learners can do mathematics to any level. Research from neuroscience shows that our brains change throughout life and no one has a “non-maths” brain. Moreover, mistakes in mathematics can grow our brains—struggling with challenging ideas provides opportunities for learning, and beliefs about ourselves as to whether we can or cannot do mathematics transform mathematical workings in our brains. This research suggests a fundamentally different approach to teaching and learning mathematics in schools. Currently, teachers favour the achievement of correct answers, and learners believe that errors are problematic and become afraid to make mistakes and, therefore, to learn.

Boaler, 2016 argues that mathematics is the only subject where students and mathematicians give very different answers to the question: “What is mathematics?” For mathematicians, mathematics is an exciting, creative endeavor, where problem solving, curiosity, excitement, intuition and perseverance play important roles. For school and even undergraduate mathematics students,

these aspects of mathematics are often not experienced and remain opaque – with students believing that mathematics is a set of procedures to be followed, which only particularly gifted people can do and understand. So how mathematics is usually taught does not provide opportunities for accessing mathematical knowledge nor for students to identify with mathematics and aspire to STEM careers.

Research from Education suggests that parents' and teachers' beliefs about mathematics have a large influence on children. In a study with Grade 1 and 2 children and parents, Maloney et. al. showed that when parents with mathematics anxiety frequently help their children with homework, their children learn less mathematics and develop more mathematics anxiety. Maloney et. al. (2015) talk about the “intergenerational transmission of low mathematics achievement and high mathematics anxiety” (p. 1480). Research in wealthier countries has shown that teachers' beliefs play a significant role in learners' achievement in and dispositions toward mathematics. For example, a study in the US showed that the more a field attributes success to giftedness rather than effort, the fewer the female and black academics are in that field because stereotypes about who belongs in the field are perpetuated. The same study found that mathematics professors hold the most fixed ideas about giftedness.

Many black, female and poor learners are excluded from mathematics, as are many white, male and richer learners. In fact, worldwide, very few learners succeed at and enjoy mathematics. Given this scenario, how do we develop teaching approaches that are inclusive, that show all learners that they can enjoy and be successful in mathematics? This is particularly important in Africa, given our socio-economic developmental needs. However, given some of the research I have quoted above, it is relevant everywhere.

The rest of the paper, presents results from two research projects: the first is about teacher development in relation to working with learners' mathematical errors and the second, about mathematical identity. Neither of the two projects has a specific focus on gender, but both focus on improving all learners' access to mathematics through improving teachers practices and approaches to teaching and to learners.

The main premise of the teacher development project was that teachers working together provide a sustainable method to support shifts in practices, towards learner participation and reasoning in mathematics. The project showed that a number of teachers did, in fact, shift their practices to include learners. The project on identity came about because we saw, through the teacher development project, that for sustainable changes, shifts in teachers' dispositions are as important as shifts in their practices.

Teacher Development for Inclusion: Working with Learner Errors

The Data Informed Practice Improvement Project

The Data Informed Practice Improvement Project (DIPIP) worked with high school mathematics teachers in Professional Learning Communities (PLCs) to support their joint learning about teaching and learning mathematics. PLCs are groups of teachers who come together to engage in regular, systematic and sustained cycles of inquiry-based learning and provide spaces where teachers can reflect and learn deliberately and systematically together, to facilitate collective and sustainable shifts in their practice. PLCs aim to establish school cultures that are conducive to ongoing learning and the development of learners, teachers and schools. These aspects of PLCs suggest that they can be useful mechanisms for sustainable teacher professional development, particularly where teacher development is seen as requiring ongoing interpretation and re-interpretation by teachers in relation to their local contexts rather than once-off, fragmented inputs by outsiders.

The focus of inquiry in the DIPIP project was learners' mathematical errors, particularly the reasoning underlying these errors. The assumption, based on the substantial errors and misconceptions research in mathematics, is that systematic errors arise from partially valid mathematical reasoning and that

making that reasoning explicit for teachers can help them to value learners' current mathematical thinking and develop more sophisticated mathematical ideas. The focus on errors was a mechanism to access three important elements of teaching and learning mathematics: how learners' thinking makes sense to them and can be worked with, even (and especially) when partially correct; how teaching practice can shift to take account of learners' errors and thinking; and teachers' content and pedagogical content knowledge.

Supporting teachers to work with learner errors in PLCs engages a number of principles of inclusive teaching and some of the research from neuroscience. It shows teachers that almost all mathematical errors are valid, have some element of reasonable thinking and can be worked with and built on. Although many mathematics teachers accept this idea at face value, it is a difficult idea to work with in practice, because teachers tend to promote correct answers in their classrooms, and errors are often seen as problems to be corrected. We worked with the notion that errors are opportunities to be embraced and which can be inclusive of more learners.

The PLCs participated in a sequence of developmental activities in which the teachers analysed learners' errors in different teaching contexts. The activities were test analysis, learner interviews, curriculum mapping, choosing "leverage" concepts, readings and discussion, planning lessons together, teaching the planned lessons, and videotaping and reflecting on the lessons together. Although the activities were set up before the project started, we built in areas of choice and flexibility for PLCs, including which mathematics content to work on, based on their analyses of learner errors in their schools.

The tests that were analysed were international (tests), national (tests) and teacher-set tests, depending on the needs and interest of the PLC. The test analysis provided an overview of strengths and weaknesses in learners' mathematical knowledge in a particular school or class. Based on the test analysis, teachers chose learners who had made interesting errors that they wanted to understand more deeply and interviewed these learners. They then took the results of these two analyses and mapped them against the curriculum, working out where the key concepts were taught and what curricular issues might have contributed to the errors. Based on these three activities, teachers chose a leverage concept, which is a concept that underlies many of the errors that learners made in a topic, for example, the equal sign and the differences between equations, expressions and formulae. Once a concept was chosen, the DIPIP facilitator found literature on that concept, including learner errors in respect of the concept. The PLC read and discussed these papers and drew on these discussions to plan lessons together, which aimed to surface learner errors in the topic and to find ways to engage them, rather than to avoid them. These lessons were taught and videotaped and the community then reflected on episodes in each teacher's lessons in order to understand their strengths and challenges in dealing with learner errors in class. In some years, communities changed the order of activities or emphasized some more than others.

Over the four years of the project, 12 schools and 50 teachers participated consistently. For at least three years, 22 teachers from six schools in four communities (one community was made up of three neighbouring schools). The PLCs met weekly during school terms, after the school day had ended. This required substantial commitment from the teachers some found it difficult to sustain.

Researching the DIPIP Project

The project developed a number of research focuses, including how the teachers' practices and knowledge shifted over time, how the communities interacted and what distinguished teachers and schools who stayed with the project for 3-4 years from those who left after shorter periods. In this paper I will focus on how teachers shifted their practices to take account of learner errors and learner thinking and to include more learners in classroom activities.

A total of 223 lessons from 19 teachers over four years, totaling more than 150 hours of lessons, were analysed using the Mathematics Quality of Instruction (MQI) instrument. The MQI instrument has five dimensions: Mode of Instruction (MI), Richness of mathematics (RM), Working with students and mathematics (WSM), Errors and imprecision (EI) and student participation in meaning making and reasoning (SMR). Each dimension was recorded at one of three levels: low, medium or high. We allocated numbers 1, 2 and 3 to each level, scored eight-minute episodes in each lesson, averaged across the episodes in each year for each teacher, calculated differences for each teacher across the years and then calculated averages for each PLC. Since there were large differences across PLCs, we reported on each PLC separately. The requirements to reach level 3 were high, and only a few teachers accomplished this a few times, so we took a shift of 0.5 as a large shift in practice.

Community 1 made mid-level changes in two dimensions from 2011 to 2012: Working with students and mathematics (0.31) and student participation in meaning-making and reasoning (0.17). Both shifts declined in 2013. The shifts could be accounted for by two of five teachers, both of whom made major changes to their teaching in 2012, and both of whom participated much less in the community in 2013, one because of illness. The other teachers' practices remained fairly stable over the four-year period. Community 2 made changes from 2012-2013 (they started the project in 2012) for three dimensions as follows: mode of instruction (0.23), richness of mathematics (0.39) and working with students and mathematics (0.45) and in 2013-2014 in four dimensions as follows: mode of instruction (0.41), richness of mathematics (0.39), working with students and mathematics (0.30) and student participation in meaning-making and reasoning (0.49). The changes were accounted for by six out of the nine teachers shifting in some or all of the dimensions over the three years, with three teachers not making major changes. In Community 3, we saw changes as follows from 2012 to 2013: mode of instruction (0.58), richness of mathematics (0.96), working with students and mathematics (1.58) and student participation in meaning-making and reasoning (1.7). So, overall we saw sustained shifts for about half of the teachers. It should be noted that very few projects have looked at teacher change in relation to PLCs and those that have, have shown modest shifts in practice, only some of which are sustained, results which are consistent with our study.

In our analyses of teacher conversations, we saw that different activities supported different kinds of conversations and that between 20% and 33% of the time was spent on content knowledge. The latter finding was important because our project did not take a direct focus on content knowledge but we found that a focus on errors does support teacher talk about their own mathematics content knowledge. One element that we worked hard to avoid was the issue of blame. Usually teachers blame learners for their errors, saying things like: they did not understand, they did not work hard enough, they aren't careful enough, or even: they are lazy, they don't get any help from home. When we moved the focus from what is wrong in errors to what is correct, we hoped that blame would be removed from learners, and that teachers would come to see making errors as a normal part of learning mathematics. This did not happen as much as we would have liked and a number of the conversations in the PLCs still showed some blaming of learners for their errors. This finding led us to think about how teachers see learners and to develop a project on mathematics identity.

Researching Identity in Mathematics Classrooms

In our research (Gardee and Brodie, in preparation), we look at teacher and learner relationships with each other and with mathematics through the lens of identity. Identity is a useful lens because it brings together the cognitive and affective dimensions of learning and becoming as well as the

personal and social aspects of both cognition and affect. It also allows us to explore teachers' practices and their dispositions simultaneously. Our working definition of identity is how people see themselves and how they are seen by others as mathematics learners and how their views of themselves and mathematics influences their practices in mathematics . We access learners' identities by observing their participation in class and by talking to them about their experiences in mathematics in and outside of school and about how mathematics figures into their plans for the future. We access teachers' views of learners by observing their pedagogical practices and their social relationships with learners and by talking to them about these.

We have developed a framework to study identity (Figure 1), which posits a relationship between personal and social identity and agency. It should be noted that social structures and broader cultural values play a role in social identities, which include how people are positioned by gender, race, poverty and inequality. While personal and social identities interact to form a mathematical identity, learners' agency is crucial in navigating between personal and social identity (Gardee and Brodie, in preparation). Figure 1 is further elaborated through the notion of offered and constructed identities (Table 1), where we see how learners' social and personal identities are affected by what teachers offer them in the classroom, and how learners' agency mediates between what is offered and what is constructed.

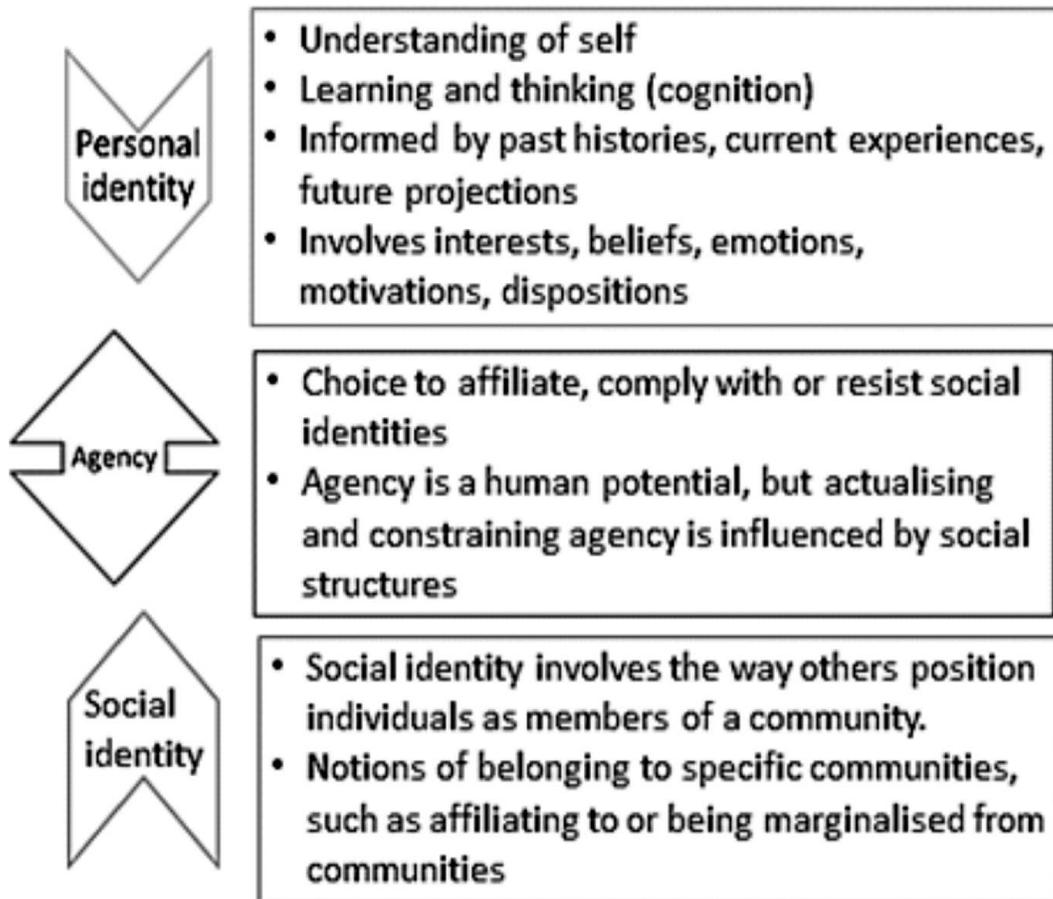


Figure 1: Mathematical Identity

Table 1: Identities Offered and Constructed

Identity offered	Identity constructed
Affiliation or partial affiliation with the classroom community	Affiliation with the classroom community
	Compliance with the classroom community
	Resistance to the classroom community
Marginalisation from the classroom community	Marginalisation from the classroom community
	Compliance with the classroom community
	Resistance to the classroom community

Classrooms where learners are offered opportunities for affiliation are those where teachers emphasise mathematical ideas and connections between them and encourage learners to both do mathematics and communicate mathematics. Teachers allow for different methods and ideas and see mistakes as opportunities for learning, have high regard for learners and believe that all learners can succeed. In these classrooms, teachers emphasise that effort, rather than ability, is needed to be successful at learning mathematics

In some classrooms, all learners are offered opportunities to construct their social identities in affiliation with the classroom community. In other classrooms, only some learners are offered opportunities for affiliation while other learners are marginalised. Research shows that learners are typically marginalised from classrooms where teachers see success in mathematics as following single procedures taught by the teacher, with no option for different ideas, and attribute success to ability, creating the illusion that not all learners can be successful learners of mathematics. Success in these classrooms is attributed to requiring some form of ability, 'specialness' or 'gift', rather than hard work .

The first quote below comes from a teacher who offer identities of affiliation to all learners in his class and who teaches in ways which encourage learners to persevere, to make errors and to learn from their errors. The second two quotes come from a teacher who believes that learners must repeat and apply mathematical principles; only some are able to do this, and some will never be able to do it. This teacher offers identities of marginalisation to most learners in his class and identities of affiliation to only very few high achievers. As shown by the learners' quotes below, he does not support students who try and make errors.

Teacher 1

“It is very crucial to give learners an opportunity to work out the problems themselves. To find out what is problematic and to try and find solutions. For me what is important is for you to see the challenge and your mind expands if you face an obstacle.”

Teacher 2

“If it is maths principles, you just need to know them because maths principles are the ones that govern you. There is nothing new and everything has been discovered.”

“Because when God probably created people, we know in science there are right people and there are left people. There are people that are good in numbers. There are people who are not good in numbers. There are people who are good in figures. There are people who are not good in figures. That is the way it is.”

Learners in Teacher 2's Class

Jack: "You tried doing your work and then like you get something wrong. That is when he will tell you like what does he say again? Hm... oh he tells you sometimes, he tells you what you wrote here is the same thing as writing rubbish."

Lane: "He just doesn't care ... whether you understand it or not... cos I've asked him many times to help me. He said no."

Senzo: "He gives you a sum, you try it then he comes and checks and tells you that that is rubbish. You know that is demotivating. You know he doesn't really care."

Learners do not always construct the identities that are offered to them. Even if learners are offered identities of affiliation, they may not affiliate with the classroom community but may construct identities of compliance or resistance. Learners comply by participating when called on by teachers without much meaningful engagement, but last to satisfy the teacher, parents or job requirements. Learners resist by choosing not to participate in the classroom or choose alternative ways of participating or disrupting the class. Similarly, learners who are offered identities of marginalisation can use their agency to construct different identities. Learners can construct their mathematical identities in compliance with the identity of marginalisation offered by choosing not to participate much with the teacher. However they may try and access help from other sources, perhaps because of their personal identity as they may still enjoy learning mathematics and wish to pass it. Learners can also construct their mathematical identities in resistance to the identity of marginalisation offered by agentively participating and attempting to develop their social identities as full members of the community. These learners' personal identities may be invested in enjoying mathematics. The learners may feel the need to be members of their classroom community, perhaps because they enjoy mathematics or would like to prove to themselves or others that they are capable of learning mathematics and being members of the classroom community. In order to construct such different identities in the face of marginalisation by the teacher, learners need considerable external resources.

We interviewed and observed learners in Grades 9 and 10 and developed a set of parameters which gave us indicators of their identities. Table 2 shows two high-achieving learners, Jack and Shane, offered identities of affiliation by both their Grade 9 and 10 teachers. Table 3 shows two low-achieving learners, both offered identities of affiliation by their Grade 9 teacher and identities of marginalisation by their Grade 10 teacher. In both classes, Shane constructed an identity of affiliation. Even though he recognised that the Grade 10 teacher was not as good for all learners as the Grade 9 teacher, he continued to draw on external support, view mathematics as useful and do well in mathematics. Jack, who constructed an identity of affiliation in Grade 9, constructed an identity of compliance in Grade 10 and no longer participated in mathematics learning in class, no longer saw mathematics as useful and was not achieving as well. A big issue for both of these high achieving learners was how the Grade 10 teacher treated other learners - they wanted to see their teacher care about all learners, not only the high achievers.

Table 2: Constructed Identities: Jack and Shane

Parameter	Jack		Shane	
	Grade 9	Grade 10	Grade 9	Grade 10
Teacher pedagogy	Used different methods to help them understand	Did not explain or use different methods	Used different methods. Made sure they understood	Did not explain
Teacher: social relationships	Made all learners feel comfortable	Was insulting	Cared about all learners	Treated him differently because he was a high achiever
Family support	None	None	Support and assistance	Support and assistance
Agency	Always participated in class. Looked for help outside of school	No longer participated or looked for outside help.	Always participated and got help from family	Always participated and got help from family
Marks	Distinctions	Reduced rapidly	Distinction	Reduced, but still performing well.
Future	Mathematically-oriented degree	No longer interested in mathematics	Mathematically-oriented degree	Mathematically-oriented degree

Table 3 shows how Lane and Senzo constructed identities of affiliation in grade 9, because of how the teacher taught them. They participated in class and out of class and their marks were improving and they were thinking of pursuing mathematics in their further studies. In grade 10 they were offered identities of marginalisation, and while Lane constructed an identity of compliance, participating in class in limited ways and thinking about choosing a career that did not involve mathematics, Senzo constructed an identity of marginalisation, completely disengaging from mathematics in class and in his vision for the future. Both learners' results reduced rapidly.

Table 3: Constructed Identities: Lane and Senzo

Parameter	Lane		Senzo	
	Grade 9	Grade 10	Grade 9	Grade 10
Teacher pedagogy	Explained well and asked a lot of questions	Did not explain. Moved too fast	Helpful and made sure they understood	Did not explain well
Teacher: social relationships	Cared about them, was motivating and made them feel comfortable	Did not care. He favoured some learners and was insulting	Helpful, caring, motivating, treated all learners equally.	Insulting, favoured high achievers, ignored him.
Family support	None	None	Parents concerned	Parents concerned
Agency	Participated in and out of school.	Limited participation	Participated in and out of school.	No longer participated
Marks	Improving steadily	Reduced rapidly	Improving steadily	Reduced rapidly
Future	Mathematically-oriented degree	May change career choice	Mathematically-oriented degree	No longer wished to pursue mathematics

The results above are representative of the bigger sample of 5 teachers and 19 learners. Two teachers, one in Grade 9 and one in Grade 10 offered identities of affiliation to all learners, and learners in these classrooms constructed identities of affiliation and compliance. Three teachers, one in Grade 9 and two in Grade 10, offered identities of affiliation to some learners and marginalisation to others. Learners in these classes constructed identities of affiliation, compliance and marginalisation, based on their own agency. So, while teachers are central in learners' construction of identities, this construction does depend on learner agency and how they integrate their personal and social identities, as they make sense of their own lives in relation to their experiences in school and out of school.

An important point from our data is that the teachers who offered identities of marginalisation to some learners were often insulting and rude to these learners, meaning that there was no or little ethic of care in their classrooms. All of the learners commented on this. They wanted to feel valued by and cared for by their teachers – both as mathematics learners and as human beings, and they wanted all learners to be treated with care – not only themselves. It is this care – how we treat all learners - that may make a big difference to the experiences of girls in schools and the larger in society. It is interesting that the learners quoted above are boys and the teachers are male, and yet they want to be cared for and valued, as do all human beings.

Conclusion

I have argued that the best way to deal with gender disparities and low achievement among all learners is to work with teachers to shift both their practices and how they see learners as mathematics learners, thus influencing the identities that learners construct. I have shown that teachers can understand learner errors as positive opportunities for learning and can shift their practices to include more learners in reasoning and meaning-making, through working on these aspects of their practice in professional learning communities, as a sustained and sustainable approach to teacher development.

However, shifting these practices may not be enough, because learners need to be accepted as mathematics learners and they need to see all learners accepted, valued and cared for. Shifts in practice are difficult to make and require considerable support. Shifts in identity, and how teachers see learners are even harder to make than changes in practice, as they both reflect and influence the broader culture and society and will require much bigger changes in schools and in all of our broader societies.

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